CLASS 9 · CHAPTER 3 · EXERCISE 3.1

Sets and Functions
Exercise 3.1 - Super Easy

Very short lines. Very small steps.

A new chapter, but the same slow way.

One small idea at a time. You can do this.



How to read this book

Read **one box**.

Stop.

Look at the **example**.

Say the steps **out loud**.

Then read the next box.

In this book you will learn

- Question 1 - write sets in **set builder notation**
- Question 2 - write sets in **tabular form**
- Question 3 - **proper subsets**
- Questions 4 and 5 - the **empty set**, and one tricky difference
- Questions 6 and 7 - the **power set**

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First: What is a Set?

A set is a group of clear, different things.

The things in a set are called its **elements**.
We write a set inside curly brackets: { }
Example: the set of vowels is {a, e, i, o, u}.

Three ways to write a set

1. **Descriptive form** - say it in words.
   The set of the first five natural numbers.
2. **Tabular form** - list every element.
   {1, 2, 3, 4, 5}
3. **Set builder notation** - give the rule.
   $\{x \mid x \in N \wedge x \leq 5\}$

$$\{x \mid \text{rule for } x\}$$

Read the rule like this: "x, such that..."
The line | means "**such that**".
The sign $\in$ means "**belongs to**".
The sign $\wedge$ means "**and**".

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Toolbox: The Number Sets

These letters come again and again. Learn them slowly.

Letter	Name	It means
N	Natural numbers	{1, 2, 3, 4, ...}
W	Whole numbers	{0, 1, 2, 3, ...}
Z	Integers	{..., -2, -1, 0, 1, 2, ...}
P	Prime numbers	{2, 3, 5, 7, 11, ...}
Q	Rational numbers	numbers like 1/2, 3/4, -5, 0.25
R	Real numbers	all numbers on the number line

Watch out!

W starts from **0**. **N** starts from **1**.
1 is NOT a prime number. The first prime is 2.
Z has minus numbers too. **N** and **W** do not.

Easy way to remember:

N is the counting numbers: 1, 2, 3...
Add 0 and you get W.
Add the minus numbers and you get Z.

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Toolbox: Subset and Power Set

Subset

A small set made from a big set.
Every element of the small set is also in the big set.
Example: $\{1, 2\}$ is a subset of $\{1, 2, 3\}$.

Proper subset

A subset that is **smaller** than the full set.
It must **miss at least one** element.
So $\{1, 2\}$ is a proper subset of $\{1, 2, 3\}$.
But $\{1, 2, 3\}$ is **not** a proper subset of itself.

Empty set

A set with **no elements at all**.
We write it as $\{\ \}$ or $\emptyset$.
It is a subset of **every** set.

Power set

The power set of A is the set of **all** its subsets.
We write it as $P(A)$.

$$\text{Number of elements of } P(A) = 2^n$$

Here n is the number of elements in A.
Count the elements. Then double 2 that many times.

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Question 1 - Set Builder Notation

Write each set in set builder notation. First find the **pattern**. Then write the **rule**.

(i). $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$

1. Look: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$.
2. So every number is a **square**: $x = n^2$.
3. Last one: $484 = 22^2$. So n goes from 1 to 22.

Answer: $\{x \mid x = n^2,\ n \in N \wedge 1 \leq n \leq 22\}$

(ii). $\{2, 4, 8, 16, \ldots, 256\}$

1. Look: $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$.
2. So every number is a **power of 2**: $x = 2^n$.
3. Last one: $256 = 2^8$. So n goes from 1 to 8.

Answer: $\{x \mid x = 2^n,\ n \in N \wedge 1 \leq n \leq 8\}$

(iii). $\{0, \pm 1, \pm 2, \ldots, \pm 1000\}$

1. These are **all the integers**, from -1000 up to 1000.
2. Integers are the set **Z**. No rule for x is needed, only the start and the end.

Answer: $\{x \mid x \in Z \wedge -1000 \leq x \leq 1000\}$

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Question 1 - continued

(iv). $\{6, 12, 18, \ldots, 120\}$

1. Look: 6, 12, 18... These are the **multiples of 6**.
2. So $x = 6n$. First: $6 = 6 \times 1$. Last: $120 = 6 \times 20$.
3. So n goes from 1 to 20.

Answer: $\{x \mid x = 6n,\ n \in N \wedge 1 \leq n \leq 20\}$

(v). $\{100, 102, 104, \ldots, 400\}$

1. The numbers go up by **2** each time, starting at 100.
2. Start at 100 and add $2n$: $x = 100 + 2n$.
3. Start: $n = 0$ gives 100. End: $100 + 2n = 400$, so $n = 150$.
4. n starts at 0, so n is in **W**, not N.

Answer: $\{x \mid x = 100 + 2n,\ n \in W \wedge 0 \leq n \leq 150\}$

(vi). $\{1, 3, 9, 27, 81, \ldots\}$

1. Look: $1 = 3^0$, $3 = 3^1$, $9 = 3^2$, $27 = 3^3$.
2. So every number is a **power of 3**: $x = 3^n$.
3. The dots mean it never stops. And n starts at 0, so $n \in W$.

Answer: $\{x \mid x = 3^n,\ n \in W\}$

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Question 1 - last part

(vii). {1, 2, 4, 5, 10, 20, 25, 50, 100}

1. Try dividing 100 by each number: 100 ÷ 1, 100 ÷ 2, 100 ÷ 4, 100 ÷ 5...
2. Every one divides 100 with **nothing left over**.
3. So these are the **divisors of 100**. Here we write the rule in words.

Answer: $\{x \mid x \text{ is a divisor of } 100,\ x \in N \wedge 1 \leq x \leq 100\}$

(viii). {5, 10, 15, ... , 100}

1. Look: 5, 10, 15... These are the **multiples of 5**.
2. So $x = 5n$. First: $5 = 5 \times 1$. Last: $100 = 5 \times 20$.

Answer: $\{x \mid x = 5n,\ n \in N \wedge 1 \leq n \leq 20\}$

(ix). The set of all integers between -100 and 1000

1. This one is given in **words**. We still write the same rule.
2. Integers means $x \in Z$.
3. **Between** means the end numbers are **not** counted. So we use $<$, not $\leq$.

Answer: $\{x \mid x \in Z \wedge -100 < x < 1000\}$

The pattern trick: always ask three things.
What is the **rule**? Where does it **start**? Where does it **stop**?

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Question 2 - Tabular Form

Now go the other way. Read the **rule**, then **list** the elements inside curly brackets.

(i). $\{x \mid x \text{ is a multiple of } 3 \wedge x \leq 35\}$

1. Write the multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33.
2. Stop! The next one, 36, is **more than 35**. Do not write it.

Answer: {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33}

(ii). $\{x \mid x \in R \wedge 2x + 1 = 0\}$

1. Solve the equation: $2x + 1 = 0 \rightarrow 2x = -1 \rightarrow x = -1/2$.
2. $-1/2$ is a real number, so it stays. Only **one** element.

Answer: {-1/2}

(iii). $\{x \mid x \in P \wedge x < 12\}$

1. P means **prime numbers**: 2, 3, 5, 7, 11, 13...
2. Keep only the ones **less than 12**. Stop before 13.

Answer: {2, 3, 5, 7, 11}

(iv). $\{x \mid x \text{ is a divisor of } 128\}$

1. Find every number that divides 128 exactly: 1, 2, 4, 8, 16, 32, 64, 128.
2. Each one is **double** the one before. Do not miss any.

Answer: {1, 2, 4, 8, 16, 32, 64, 128}

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Question 2 - last part

(v). $\{x \mid x = 2^n,\ n \in N \wedge n < 8\}$

1. n can be 1, 2, 3, 4, 5, 6, 7. Not 8, because $n < 8$.
2. Now put each n in: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$.

Answer: {2, 4, 8, 16, 32, 64, 128}

(vi). $\{x \mid x \in N \wedge x + 4 = 0\}$

1. Solve: $x + 4 = 0 \rightarrow x = -4$.
2. But wait. Is -4 a **natural** number? **No!** N has no minus numbers.
3. So **nothing** fits the rule. The set is empty.

Answer: { } the empty set

(vii). $\{x \mid x \in N \wedge x = x\}$

1. Every number is equal to itself. So **every** natural number fits.
2. The list never ends, so we end it with dots.

Answer: {1, 2, 3, 4, ... }

(viii). $\{x \mid x \in Z \wedge 3x + 1 = 0\}$

1. Solve: $3x + 1 = 0 \rightarrow 3x = -1 \rightarrow x = -1/3$.
2. Is -1/3 an **integer**? **No!** Integers have no fractions.
3. So again, nothing fits the rule.

Answer: { } the empty set

Big trap in this question!

Solving the equation is only **half** the work.
Always check: does the answer **belong** to the set named in the rule?

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Question 3 - Two Proper Subsets

Write **two** proper subsets of each set. A proper subset is **smaller** - it must miss at least one element. Many answers are possible. These are just good ones.

(i). {a, b, c}

1. Take just one letter at a time. A single element is always a proper subset.

Answer: {a} and {b}

(ii). {0, 1}

1. Same idea. Take one number at a time.

Answer: {0} and {1}

(iii). N {1, 2, 3, 4, ...}

1. N is very big, so we can take a whole **family** of numbers from inside it.
2. The prime numbers are all in N, but they are not all of N.
3. The even natural numbers are also all in N, but they are not all of N.

Answer: The set of prime numbers and the set of even natural numbers

(iv). Z {..., -1, 0, 1, ...}

1. Look for families that live inside the integers.
2. Every prime number is an integer. Every whole number is an integer too.

Answer: The set of prime numbers and the set of whole numbers

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Question 3 - last part

(v). Q the rational numbers

1. Every integer can be written as a fraction, like 3 = 3/1. So the integers live inside Q.
2. Every natural number is also a rational number.

Answer: The set of integers and the set of natural numbers

(vi). R the real numbers

1. R is the biggest family here. It holds the rationals **and** the irrationals.
2. Each family on its own is smaller than all of R.

Answer: The set of rational numbers and the set of irrational numbers

(vii). $\{x / x \in Q \wedge 0 < x \leq 2\}$

1. We need rational numbers **greater than 0** and **up to 2**.
2. Pick two simple fractions that fit: 1/2 fits, and 3/4 fits.

Answer: {1/2} and {3/4}

Question 4 - A Set With No Proper Subset

Question: Is there any set which has no proper subset? If so, name that set.

1. A proper subset must be **smaller** than the set.
2. The **empty set** has no elements at all. Nothing can be smaller than nothing.
3. So it has **no** proper subset.

Answer: Yes. The empty set, { }

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Question 5 - Spot the Difference

Question: What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

1 Count the elements. That is the whole trick.

$\{a, b\}$

This set has **two** elements:
a and **b**.
They are separate, side by side.

$\{\{a, b\}\}$

This set has only **one** element:
the set **$\{a, b\}$** itself.
The inner set is packed inside, like a box inside a box.

Answer: $\{a, b\}$ has 2 elements. $\{\{a, b\}\}$ has 1 element.

Question 6 - How Big is the Power Set?

Use the rule: **number of elements of $P(A) = 2^n$**, where n is the number of elements in the set.

(i). $\{\}$

1 The empty set has $n = 0$ elements. $2^0 = 1$.

Answer: 1 element

(ii). $\{0, 1\}$

1 Count: $n = 2$. $2^2 = 2 \times 2 = 4$.

Answer: 4 elements

(iii). $\{1, 2, 3, 4, 5, 6, 7\}$

1 Count: $n = 7$. $2^7 = 128$.

Answer: 128 elements

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Question 6 - last part

(iv). {0, 1, 2, 3, 4, 5, 6, 7}

1. Count carefully: 0, 1, 2, 3, 4, 5, 6, 7 - that is $n = 8$, not 7. Zero counts too!
2. $2^8 = 256$.

Answer: 256 elements

(v). {a, {b, c}}

1. Count the elements: **a** is one element. **{b, c}** is the second element.
2. A set inside a set counts as **one** element. So $n = 2$.
3. $2^2 = 4$.

Answer: 4 elements

(vi). {{a, b}, {b, c}, {d, e}}

1. Each small set is **one** element: {a, b}, then {b, c}, then {d, e}.
2. So $n = 3$. $2^3 = 2 \times 2 \times 2 = 8$.

Answer: 8 elements

The trap in (v) and (vi)

Do **not** count the letters inside the small sets.
Count only the things separated by commas in the **outer** set.
{a, {b, c}} has **2** elements, not 3.

Doubling table - learn this by heart

$2^0 = 1$ · $2^1 = 2$ · $2^2 = 4$ · $2^3 = 8$ · $2^4 = 16$
$2^5 = 32$ · $2^6 = 64$ · $2^7 = 128$ · $2^8 = 256$

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Question 7 - Write the Power Set

*Now write **all** the subsets, not just count them. Work in order: first the empty set, then one element at a time, then two together, then the whole set.*

(i). {9, 11} Let A = {9, 11}

1. First check the count: n = 2, so P(A) must have $2^2 = 4$ subsets.
2. No elements: ∅
3. One element: {9}, {11}
4. Both elements: {9, 11}

Answer: P(A) = {∅, {9}, {11}, {9, 11}}

(ii). {+, −, ×, ÷} Let A = {+, −, ×, ÷}

1. Count: n = 4, so P(A) must have $2^4 = 16$ subsets. Work slowly, in groups.
2. **None:** ∅
3. **One at a time (4):** {+}, {−}, {×}, {÷}
4. **Two at a time (6):** {+, −}, {+, ×}, {+, ÷}, {−, ×}, {−, ÷}, {×, ÷}
5. **Three at a time (4):** {+, −, ×}, {+, −, ÷}, {+, ×, ÷}, {−, ×, ÷}
6. **All four (1):** {+, −, ×, ÷}
7. Add them up: 1 + 4 + 6 + 4 + 1 = **16**. Perfect, none missing.

Answer: P(A) has all 16 subsets listed above

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Question 7 - last part

(iii). $\{\emptyset\}$ Let $A = \{\emptyset\}$

1. Careful! This set is **not** empty. It has **one** element: the empty set itself.
2. So $n = 1$, and $P(A)$ has $2^1 = \mathbf{2}$ subsets.
3. First subset: the empty set, $\emptyset$. Second subset: the whole set, $\{\emptyset\}$.

Answer: $P(A) = \{\emptyset, \{\emptyset\}\}$

(iv). $\{a, \{b, c\}\}$ Let $A = \{a, \{b, c\}\}$

1. Count: **a** is one element, **{b, c}** is one element. So $n = 2$.
2. $P(A)$ must have $2^2 = \mathbf{4}$ subsets.
3. None: $\emptyset$ One at a time: $\{a\}$, and $\{\{b, c\}\}$ - note the **double** brackets.
4. Both together: $\{a, \{b, c\}\}$ - this is A itself.

Answer: $P(A) = \{\emptyset, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

The hardest idea in this exercise

$\emptyset$ means the empty set. It has **0** elements.
$\{\emptyset\}$ means a set holding the empty set. It has **1** element.
Think of it like this: an empty bag is still **one bag**.

Always check your power set:
Count your subsets when you finish.
The count must be $\mathbf{2^n}$.
If it is not, one subset is missing. Find it.

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Check Your Answers

Cover the page. Try the question. Then look here.

Question	Answer
Q1 Set builder	(i) $\{x \mid x = n^2, 1 \le n \le 22\}$ (ii) $\{x \mid x = 2^n, 1 \le n \le 8\}$ (iii) $\{x \mid x \in \mathbb{Z}, -1000 \le x \le 1000\}$ (iv) $\{x \mid x = 6n, 1 \le n \le 20\}$ (v) $\{x \mid x = 100 + 2n, n \in \mathbb{W}, 0 \le n \le 150\}$ (vi) $\{x \mid x = 3^n, n \in \mathbb{W}\}$ (vii) $\{x \mid x$ is a divisor of $100\}$ (viii) $\{x \mid x = 5n, 1 \le n \le 20\}$ (ix) $\{x \mid x \in \mathbb{Z}, -100 < x < 1000\}$
Q2 Tabular form	(i) {3, 6, 9, …, 33} (ii) {-1/2} (iii) {2, 3, 5, 7, 11} (iv) {1, 2, 4, 8, 16, 32, 64, 128} (v) {2, 4, 8, 16, 32, 64, 128} (vi) { } (vii) {1, 2, 3, 4, …} (viii) { }
Q3 Proper subsets	(i) {a}, {b} (ii) {0}, {1} (iii) primes, even naturals (iv) primes, whole numbers (v) integers, naturals (vi) rationals, irrationals (vii) {1/2}, {3/4}
Q4	Yes - the empty set { } has no proper subset
Q5	{a, b} has 2 elements; {{a, b}} has 1 element
Q6 Size of power set	(i) 1 (ii) 4 (iii) 128 (iv) 256 (v) 4 (vi) 8
Q7 Power set	(i) {∅, {9}, {11}, {9, 11}} (ii) 16 subsets of {+, −, ×, ÷} (iii) {∅, {∅}} (iv) {∅, {a}, {{b, c}}, {a, {b, c}}}

Three things to remember

1. Find the **pattern** first, then write the rule.
2. After solving, check the answer **belongs** to the named set.
3. Power set size is always 2^n. Count n with care.

Exercise 3.1 - done. Well done, you finished Chapter 3's first exercise.

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