CLASS 9 · CHAPTER 2 · EXERCISE 2.4

Logarithms Super Easy

Very short lines. Very small steps.

Read slow. Take your time. You can do this.



How to read this book

*Read **one box**.*
Stop.
*Look at the **example**.*
*Say the steps **out loud**.*
Then read the next box.

In this book you will learn

- *The **4 laws of logarithms** - your toolbox*
- *Question 1 to Question 4 - laws, step by step*
- *Question 5 - log table, **Characteristic** and **Mantissa***
- *Questions 6, 7 and 8 - real-life sums*

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First: The 4 Laws

Every sum in this exercise uses these **4 laws**. Learn them first.

Law 1 – Times becomes Plus

$$\log (a \times b) = \log a + \log b$$

Two logs with **+** can join into **one log**.

One log of a **times** can split into **two logs**.

Law 2 – Divide becomes Minus

$$\log (a / b) = \log a - \log b$$

Top number first. Bottom number second.

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The 4 Laws - part 2

Law 3 - Power comes in front

$$\log a^n = n \times \log a$$

The small power **jumps down** in front.

Law 4 - Change the base

$$\log_b x = \log x / \log b$$

We need this only in Question 2 (iii) and (iv).

2 golden facts - use them again and again

$\log_a a = 1$ (same number, same base = 1)

$\log_a 1 = 0$ (log of 1 is always 0)

Say it: "Times is plus. Divide is minus. Power comes in front."

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Question 1 - Find the value. No calculator.

Trick: break big numbers into small parts. Then use the golden facts.

(i). $\log_2 18 - \log_2 9$

1. Split 18. $18 = 2 \times 9$.
2. $\log_2 (2 \times 9) - \log_2 9 = \log_2 2 + \log_2 9 - \log_2 9$ (Law 1)
3. $\log_2 9$ and $-\log_2 9$ **cut each other**. Only $\log_2 2$ is left.
4. $\log_2 2 = 1$ (same number, same base)

Answer: 1

(ii). $\log_2 64 + \log_2 2$

1. 64 is a power of 2. $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$. So $64 = 2^6$.
2. $\log_2 2^6 + \log_2 2 = 6 \log_2 2 + \log_2 2$ (Law 3)
3. $\log_2 2 = 1$. So $6(1) + 1 = 6 + 1$.

Answer: 7

(iii). $\frac{1}{3} \log_3 8 - \log_3 18$

1. $8 = 2^3$. And $18 = 2 \times 3^2$ ($2 \times 9 = 18$).
2. $\frac{1}{3} \log_3 2^3 - \log_3 (2 \times 3^2)$
3. First part: the 3 and the $\frac{1}{3}$ cut. We get $\log_3 2$.
4. Second part splits (Law 1): $\log_3 2 + 2 \log_3 3$.
5. So: $\log_3 2 - \log_3 2 - 2 \log_3 3$. The $\log_3 2$ parts **cut**.
6. Left: $-2 \log_3 3 = -2(1)$.

Answer: −2

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Question 1 – more sums

(iv). $2 \log 2 + \log 25$

No base is written. That means base 10.

1. Power goes back up (Law 3 backwards): $2 \log 2 = \log 2^2 = \log 4$.
2. Now: $\log 4 + \log 25$. Plus joins into one log (Law 1).
3. $\log (4 \times 25) = \log 100$.
4. $100 = 10^2$. So $\log 10^2 = 2 \log 10 = 2(1)$.

Answer: 2

(v). $\frac{1}{3} \log_4 64 + 2 \log_5 25$

1. $64 = 4^3$ $(4 \times 4 \times 4)$. $25 = 5^2$.
2. $\frac{1}{3} \log_4 4^3 + 2 \log_5 5^2$
3. Powers come in front (Law 3): $\log_4 4 + 2 \times 2 \log_5 5$.
4. $\log_4 4 = 1$ and $\log_5 5 = 1$. So $1 + 4(1) = 1 + 4$.

Answer: 5

(vi). $\log_3 12 + \log_3 0.25$

1. $12 = 3 \times 4$. And $0.25 = 1/4$.
2. $\log_3 3 + \log_3 4 + \log_3 (1/4)$ (Law 1 and Law 2)
3. $\log_3 (1/4) = \log_3 1 - \log_3 4$.
4. $\log_3 4$ and $- \log_3 4$ **cut**. Left: $\log_3 3 + \log_3 1 = 1 + 0$.

Answer: 1

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Question 2 - Make ONE log

*Job: join many logs into **one single log**. First push every number in front **up** as a power.*

(i). $\frac{1}{2} \log 25 + 2 \log 3$

1. ½ goes up: $\log 25^{1/2}$. 2 goes up: $\log 3^2$.
2. $25^{1/2}$ means root of 25 = 5. $3^2 = 9$.
3. $\log 5 + \log 9$. Plus joins (Law 1): $\log (5 \times 9)$.

Answer: $\log 45$

(ii). $\log 9 - \log \frac{1}{3}$

1. Minus means divide (Law 2 backwards): $\log (9 \div \frac{1}{3})$.
2. Divide by a fraction: turn it and times. $9 \times 3 = 27$.

Answer: $\log 27$

(iii). $\log_5 b^2 \times \log_a 5^3$ (note: this sum is a TIMES)

1. Powers come in front (Law 3): $2 \log_5 b \times 3 \log_a 5$.
2. Change the bases (Law 4): $\log_5 b = \log b / \log 5$. $\log_a 5 = \log 5 / \log a$.
3. $2 \times 3 \times (\log b / \log 5) \times (\log 5 / \log a)$. The log 5 parts **cut**.
4. Left: $6 \times (\log b / \log a) = 6 \log_a b$ (Law 4 backwards).

Answer: $6 \log_a b$

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Question 2 – more sums

(iv). $2 \log_3 x + \log_3 y$

1. 2 goes up as a power: $\log_3 x^2 + \log_3 y$.
2. Plus joins into one log (Law 1): the numbers times together.

Answer: $\log_3 x^2 y$

(v). $4 \log_5 x - \log_5 y + \log_5 z$

1. 4 goes up: $\log_5 x^4 - \log_5 y + \log_5 z$.
2. Plus parts go on **top**: x^4 and z. Minus part goes at the **bottom**: y.

Answer: $\log_5 (x^4 z / y)$

(vi). $2 \ln a + 3 \ln b - 4 \ln c$

ln is just a log with a special base. Same laws!

1. Numbers go up: $\ln a^2 + \ln b^3 - \ln c^4$.
2. Plus parts on top: $a^2 b^3$. Minus part at bottom: c^4.

Answer: $\ln (a^2 b^3 / c^4)$

Easy rule: Plus = top. Minus = bottom.

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Question 3 - Open the log out (Expand)

*This is Question 2 in **reverse**. One log becomes many small logs.*

(i). $\log(11/5)$

1. Divide becomes minus (Law 2): top first, bottom second.

Answer: $\log 11 - \log 5$

(ii). $\log_5 \sqrt{(8a^6)}$

1. Root sign means power ½. $8 = 2^3$. So: $\log_5 (2^3\, a^6)^{1/2}$.
2. Times the powers by ½: $3 \times ½ = 3/2$. $6 \times ½ = 3$.
3. Now: $\log_5 2^{3/2}\, a^3$. Times splits into plus (Law 1).
4. Powers come in front (Law 3).

Answer: $3/2 \log_5 2 + 3 \log_5 a$

(iii). $\ln(a^2b/c)$

1. Top part: a^2b is a times, so it splits with plus. Bottom c goes with minus.
2. $\ln a^2 + \ln b - \ln c$.
3. Power 2 comes in front: $2 \ln a$.

Answer: $2 \ln a + \ln b - \ln c$

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Question 3 – more sums

(iv). $\log (xy / z)^{1/9}$

1. Power 1/9 comes in front first (Law 3): $1/9 \log (xy / z)$.
2. Inside: x times y is plus. Divide by z is minus.

Answer: $1/9 (\log x + \log y - \log z)$

(v). $\ln \sqrt[3]{(16x^3)}$ (cube root)

1. Cube root means power ⅓. $16 = 2^4$ (2×2×2×2). So: $\ln (2^4 x^3)^{1/3}$.
2. Times the powers by ⅓: $4 \times ⅓ = 4/3$. $3 \times ⅓ = 1$.
3. Now: $\ln 2^{4/3} x$. Split the times, then bring the power in front.

Answer: $4/3 \ln 2 + \ln x$

(vi). $\log_2 ((1 - a) / b)^5$

1. Power 5 comes in front (Law 3): $5 \log_2 ((1 - a) / b)$.
2. Divide becomes minus (Law 2): top $(1 - a)$ first, bottom b second.

Answer: $5 [\log_2 (1 - a) - \log_2 b]$

Careful: $(1 - a)$ stays together in one bracket. Do not split it.

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Question 4 - Find x

The key move

If $\log_b$ (thing) = n, then the thing = b^n.

Say it like this: "**The base jumps up.**"

(i). $\log 2 + \log x = 1$

1. Join into one log (Law 1): $\log 2x = 1$.
2. Base is 10. It jumps up: $10^1 = 2x$. So $10 = 2x$.
3. Divide both sides by 2: $x = 10 / 2$.

Answer: x = 5

(ii). $\log_2 x + \log_2 8 = 5$

1. Join into one log (Law 1): $\log_2 8x = 5$.
2. Base 2 jumps up: $2^5 = 8x$. $2^5 = 32$. So $32 = 8x$.
3. $x = 32 / 8$.

Answer: x = 4

(iii). $(81)^x = (243)^{x+2}$

1. Make both sides powers of 3: $81 = 3^4$. $243 = 3^5$.
2. $(3^4)^x = (3^5)^{x+2}$ gives $3^{4x} = 3^{5x+10}$.
3. Bases are the same, so powers are equal: $4x = 5x + 10$.
4. $4x - 5x = 10$, so $-x = 10$.

Answer: x = −10

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Question 4 - more sums

(iv). $(1/27)^{x-6} = 27$

1. 1/27 is 27 with power −1. So: $(27^{-1})^{x-6} = 27^1$.
2. Left side becomes 27^{-x+6}. Bases are the same, so powers are equal.
3. $-x + 6 = 1$. So $-x = 1 - 6 = -5$.

Answer: x = 5

(v). $\log(5x - 10) = 2$

1. Base is 10. It jumps up: $10^2 = 5x - 10$. So $100 = 5x - 10$.
2. Move the 10: $100 + 10 = 5x$. So $110 = 5x$.
3. $x = 110 / 5$.

Answer: x = 22

(vi). $\log_2(x + 1) - \log_2(x - 4) = 2$

1. Minus becomes one log of a divide (Law 2): $\log_2((x+1)/(x-4)) = 2$.
2. Base 2 jumps up: $(x+1)/(x-4) = 2^2 = 4$.
3. Times both sides: $x + 1 = 4(x - 4) = 4x - 16$.
4. Bring x parts together: $4x - x = 1 + 16$. So $3x = 17$.

Answer: x = 17/3 = 5⅔

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Question 5 - Use the log table

Same 4 steps, every time

1. Call the answer **x**.
2. Take **log** of both sides. Use the laws.
3. Put the table values in. Add and subtract.
4. Take **Antilog**. That gives x.

*In the table, the whole-number part is the **Characteristic**. The decimal part is the **Mantissa**. Example: log 3.68 = 0.5658, so **Characteristic = 0**, **Mantissa = 0.5658**.*

(i). (3.68 × 4.21) / 5.234

1. Let x = (3.68 × 4.21) / 5.234. Take log of both sides.
2. log x = log 3.68 + log 4.21 − log 5.234 (times = plus, divide = minus)
3. Table values: = 0.5658 + 0.6243 − 0.7188 = **0.4713**.
4. Antilog of 0.4713: **Characteristic = 0**, **Mantissa = 0.4713**.

Answer: x = 2.9601

(ii). 4.67 × 2.11 × 2.397

1. Let x = 4.67 × 2.11 × 2.397. Take log of both sides.
2. log x = log 4.67 + log 2.11 + log 2.397 (all times, so all plus)
3. Table values: = 0.6693 + 0.3243 + 0.3797 = **1.3733**.
4. Antilog of 1.3733: **Characteristic = 1**, **Mantissa = 0.3733**.

Answer: x = 23.6194

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Question 5 - more sums

(iii). $((20.46)^2 \times 2.4122) / 754.3$

1. Let x = that whole thing. Take log of both sides.
2. log x = 2 log 20.46 + log 2.4122 − log 754.3 (power 2 comes in front)
3. Table values: = 2(1.3109) + 0.3824 − 2.8775.
4. 2(1.3109) = 2.6218. So log x = 2.6218 + 0.3824 − 2.8775 = **0.1267**.
5. Antilog of 0.1267: **Characteristic = 0**, **Mantissa = 0.1267**.

Answer: x = 1.3388

(iv). $(\sqrt[3]{9.364} \times 21.64) / 3.21$

1. Cube root = power ⅓. Let x = that whole thing. Take log of both sides.
2. log x = ⅓ log 9.364 + log 21.64 − log 3.21.
3. Table values: = ⅓(0.9715) + 1.335 − 0.5065.
4. ⅓(0.9715) = 0.3238. So log x = 0.3238 + 1.335 − 0.5065 = **1.1523**.
5. Antilog of 1.1523: **Characteristic = 1**, **Mantissa = 0.1523**.

Answer: x = 14.2003

Remember: Antilog is the way back. Log takes a number in. Antilog brings the number out.

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Question 6 - Earthquake

The size of an earthquake is **M**. The rule is:

$$M = \log_{10}(A / A_0)$$

A = amplitude = **10,000**. A_0 = reference amplitude = **10**.

1. Put the numbers in: $M = \log_{10}(10000 / 10)$.
2. $10000 / 10 = 1000$. So $M = \log_{10} 1000$.
3. $1000 = 10^3$. So $M = 3 \log_{10} 10 = 3(1)$.

Answer: M = 3

Question 7 - Money doubles

Abdullah puts **Rs. 1,00,000** in a scheme. It grows by **5%** every year.

$$y = 100000\,(1.05)^t$$

y = money after t years. Double money means $y =$ **200000**. Find **t**.

1. $200000 = 100000\,(1.05)^t$. Divide by 100000: $2 = (1.05)^t$.
2. Take log of both sides: $\log 2 = t \times \log 1.05$.
3. Table values: $0.3010 = t \times (0.0212)$.
4. $t = 0.3010 / 0.0212 = 14.19\ldots$ The book writes about 14.

Answer: t = 14 years

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Question 8 – Mountain temperature

Going up, the temperature falls by **3%** for every 100 m. That is a factor of **0.97**. At sea level it is **20°C**.

$$T = T_i \times 0.97^{h/100}$$

Height **h = 500 m**. So h/100 = **5**.

1. $T = 20 \times (0.97)^5$. Take log of both sides.
2. $\log T = \log 20 + 5 \log 0.97$.
3. Table values: $= 1.3010 + 5(-0.0132) = 1.3010 - 0.066 = \mathbf{1.235}$.
4. Antilog of 1.235: **Characteristic = 1**, **Mantissa = 0.235**.

Answer: T = 17.18 °C

Well done. You finished all 8 questions. Next page: every answer, for checking.

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Remember - check your answers

Question	Answers
Q1 Evaluate	(i) 1 (ii) 7 (iii) −2 (iv) 2 (v) 5 (vi) 1
Q2 One log	(i) log 45 (ii) log 27 (iii) 6 $\log_a b$ (iv) $\log_3 x^2y$ (v) $\log_5 (x^4z / y)$ (vi) $\ln (a^2b^3 / c^4)$
Q3 Expand	(i) log 11 − log 5 (ii) 3/2 $\log_5 2$ + 3 $\log_5 a$ (iii) 2 ln a + ln b − ln c (iv) 1/9(log x + log y − log z) (v) 4/3 ln 2 + ln x (vi) 5[$\log_2(1-a)$ − $\log_2 b$]
Q4 Find x	(i) 5 (ii) 4 (iii) −10 (iv) 5 (v) 22 (vi) 17/3 = 5⅔
Q5 Log table	(i) 2.9601 (ii) 23.6194 (iii) 1.3388 (iv) 14.2003
Q6, Q7, Q8	Q6: M = 3 Q7: 14 years Q8: 17.18 °C

✓ Times is plus. Divide is minus. Power comes in front.

✓ $\log_a a = 1$ and $\log_a 1 = 0$.

✓ Plus on top, minus at bottom, when you make one log.

✓ Log table: log first, then Antilog. **Characteristic** + **Mantissa**.

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