Logarithms — Made Easy!

Class 9 · Chapter 2 · Exercise 2.2 · Slow & Simple Notes

What is a logarithm? Just a friendly question

A logarithm (say it: LOG-a-rithm) is not a new monster. It is only a QUESTION: 'What power must I put on this base to get that number?' Example: what power of 2 gives 8? Because 2 × 2 × 2 = 8, the power is 3 — and that 3 IS the logarithm. The answer to every log is a power.

THE ONE RULE OF THIS EXERCISE — THE FLIP

$a^b = c$ is the same fact as $\log_a c = b$

- Left side: POWER form. Right side: LOG form. Same fact, two costumes — flip freely between them.
- The BASE never moves house: the base of the power is the little base of the log.
- The POWER jumps over and becomes the ANSWER of the log. The result number sits after the word log.

SAY IT OUT LOUD — THE QUESTION TRICK

'Log of 8, base 2, equals 3' is read as a question: 'What power of 2 makes 8? — 3!' Whenever a log looks scary, ask the question out loud. The answer is always just a small power.

Base stays base. Power becomes the answer.
Learn the flip — the whole exercise is yours!

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Q.1 Powers Become Logs (i–iv)

Power form becomes log form. The base stays the base.

Q.1 gives you POWER form and asks for LOG form. Every time, do the same 3 moves: (1) base stays base, (2) the result number goes after log, (3) the power becomes the answer.

(i) $10^3 = 1000$

Name the pieces: base = 10, power = 3, result = 1000. Now flip them into their log places:

Answer: $\log_{10} 1000 = 3$

(ii) $2^8 = 256$

Base = 2, power = 8, result = 256. Flip:

Answer: $\log_2 256 = 8$

(iii) $3^{-3} = 1/27$

Careful — the power is NEGATIVE: −3. A negative power always makes a small fraction (here 1/27). The flip does not change at all: that −3 simply becomes the log's answer.

Answer: $\log_3 1/27 = -3$

(iv) $20^2 = 400$

Even a big base like 20 behaves exactly the same. Base = 20, power = 2, result = 400.

Answer: $\log_{20} 400 = 2$

Self-check habit (5 seconds!): after flipping, ask 'base to the power of my answer — do I get the result?' 10 to the power 3 = 1000. This tiny check catches almost every slip.

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Q.1 The Sneaky Ones (v–viii)

Negative powers, fraction powers and one letter question.

(v) $16^{-1/4} = 1/2$

The power here is the fraction −1/4. Do not panic — nothing in the flip changes. The whole power, exactly as it is, minus sign and all, becomes the answer.

Answer: $\log_{16} 1/2 = -1/4$

(vi) $11^{2} = 121$

A calm one to rest on: base = 11, power = 2, result = 121. Flip:

Answer: $\log_{11} 121 = 2$

(vii) $p = q^{r}$

Letters instead of numbers — but the same flip! Read it as q to the power r equals p: the base is the letter q, the power is r, and the result is p. Letters are just numbers in disguise.

Answer: $\log_{q} p = r$

(viii) $32^{-1/5} = 1/2$

Base = 32, power = −1/5, result = 1/2. Flip all three pieces into their log places:

Answer: $\log_{32} 1/2 = -1/5$

Why is it true? 32 to the power 1/5 is 2, because 2 × 2 × 2 × 2 × 2 = 32. So 32 to the power −1/5 is 1/2. Fraction powers are only roots in a costume — more on that on page 5.

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Q.2 Logs Become Powers (i–iv)

Log form becomes power form — flip the rule backwards.

Q.2 is the very same flip in reverse: LOG form is given, write POWER form. Base stays base, the log's answer becomes the power, and the number becomes the result.

(i) $\log_5 125 = 3$

Name the pieces: base = 5, number = 125, answer (the future power) = 3. Flip:

Answer: $5^3 = 125$

(ii) $\log_2 16 = 4$

Base = 2, answer = 4, number = 16. Flip:

Answer: $2^4 = 16$

(iii) $\log_{23} 1 = 0$

A SPECIAL one: the number is 1, and the answer is 0. That is no accident — any base to the power 0 gives 1, so the log of 1 is ALWAYS 0, whatever the base is.

Answer: $23^0 = 1$

(iv) $\log_5 5 = 1$

Another SPECIAL one: the number EQUALS the base (5 and 5), and the answer is 1 — because a number to the power 1 is itself. The log of the base itself is ALWAYS 1.

Answer: $5^1 = 5$

Two gifts to remember forever: log of 1 = 0 (every base), and log of the base itself = 1. Examiners love both — now they are free marks for you.

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Q.2 Sneaky Ones Again (v–viii)

Negative answers, and questions written back-to-front.

(v) $\log_2 1/8 = -3$

The answer is negative (−3), so the power will be negative — and negative powers make fractions: 2 to the power −3 means 1 over 2^3, which is 1/8. Flip as usual:

Answer: $2^{-3} = 1/8$

(vi) $1/2 = \log_9 3$

The log is sitting on the WRONG side! No problem — an equals sign works both ways. First turn the sentence around: log of 3, base 9, equals 1/2. Now flip:

Answer: $9^{1/2} = 3$

Why is it true? The power 1/2 means SQUARE ROOT, and the square root of 9 is 3. Remember: power 1/2 = root in disguise.

(vii) $5 = \log_{10} 100000$

Back-to-front again: turn it around — log of 100000, base 10, equals 5 — then flip in the usual way:

Answer: $10^5 = 100000$

(viii) $\log_4 1/16 = -2$

The answer is −2, so expect a negative power — and that means a fraction result. Check afterwards: 4 to the power 2 = 16, so 4 to the power −2 = 1/16.

Answer: $4^{-2} = 1/16$

> **THE BACK-TO-FRONT TRAP**
> In (vi) and (vii) the answer is written FIRST. Do not let it confuse your flip — calmly rewrite the sentence with the log on the left, then flip as usual.

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Q.3 Find x — Hiding in 3 Places

First flip into power form — then hunt for x.

In Q.3 the letter x hides in one of three places: as the BASE, as the ANSWER (power), or as the NUMBER. The rescue is always the same first step — flip into power form, where x is easy to catch.

(i) $\log_x 64 = 3$

x is hiding as the BASE. Flip: x to the power 3 = 64. Now ask: which number, multiplied by itself 3 times, gives 64? Split 64 by dividing by 4: 64 ÷ 4 = 16, 16 ÷ 4 = 4, 4 ÷ 4 = 1 — it took three 4s, so 64 = 4 × 4 × 4.

So: $x^3 = 4^3$, $x = 4$

(ii) $\log_5 1 = x$

x is hiding as the ANSWER. Flip: 5 to the power x = 1. But 1 is the result of power 0 (anything to the power 0 is 1), so 5 to the power x = 5 to the power 0 — the powers must match:

Answer: $x = 0$

It is the golden fact from page 4 again: the log of 1 is always 0.

(iii) $\log_x 8 = 1$

x is the base, and the answer (power) is 1. Flip: x to the power 1 = 8. But power 1 changes nothing — a number to the power 1 is the number itself. So x is simply:

Answer: $x = 8$

Pattern to spot: whenever the flip gives 'something to the power 1', the answer is staring at you — no calculation at all.

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Q.3 Find x (iv–vi)

x hiding as the number — and the halving ladder for 1024.

(iv) $\log_{10} x = -3$

x is hiding as the NUMBER this time — the easiest hiding place! Flip, and the power side IS x: x = 10 to the power −3. A negative power means '1 over': 1 over 10^3, and 10^3 = 1000.

Answer: x = 1/1000

(v) $\log_4 x = 3/2$

x is the number again. Flip: x = 4 to the power 3/2. A fraction power looks scary — cross this bridge: 4 is 2 × 2, that is 2 to the power 2. So x is (2 to the power 2) to the power 3/2. When powers stack, multiply them: 2 × 3/2 = 3.

Answer: $x = 2^3 = 8$

(vi) $\log_2 1024 = x$

x is the answer. Flip: 2 to the power x = 1024. So: how many 2s multiply together to make 1024? Use the HALVING LADDER — keep halving 1024 and count the steps:

1024 – 512 – 256 – 128 – 64 – 32 – 16 – 8 – 4 – 2 – 1

Count the jumps: 10 jumps. So 1024 is ten 2s multiplied together — 2 to the power 10.

Answer: x = 10

The halving ladder is your best friend for every 'log of a big power' question: halve, halve, halve... and count. No calculator, no fear.

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1-Page Revision — The Flip Rule

Read this before the test — one rule runs the whole exercise.

1. THE FLIP: a to the power b = c means log of c (with base a) = b. Base stays base, the power becomes the answer, and the answer of a log is ALWAYS a power.
2. Q.1 direction: power form becomes log form. Q.2 direction: log form becomes power form. One flip, two directions.
3. Negative answer, fraction result: 2 to the power −3 = 1/8, so the log of 1/8 (base 2) is −3. Negative powers always shrink into fractions.
4. Fraction powers are roots in costume: power 1/2 = square root (9 to the power 1/2 = √9 = 3). Powers like −1/4 or −1/5 simply travel with the flip — copy them exactly.
5. GOLDEN FACTS: log of 1 = 0 for every base (23 to the power 0 = 1). Log of the base itself = 1 ($5^1 = 5$). Both are exam favourites.
6. FIND x: flip FIRST, think second. x as base: make both sides the same power ($x^3 = 4^3$, so x = 4). x as the number: the power side IS x. x as the answer: write the number as a power of the base (halving ladder!).
7. Back-to-front questions (answer written first, log second) — turn the sentence around, then flip. Equals works both ways.

TRY YOURSELF

A) Write $7^2 = 49$ in log form.

B) Write 'log of 81, base 3, equals 4' in power form.

C) Find x: log of x, base 2, equals 5.

D) Find x: log of 25, base x, equals 2.

Answers: A) log of 49 (base 7) = 2 B) $3^4 = 81$ C) x = 32 D) x = 5

Exercise 2.2 — done! The flip is yours now.
Next: the laws of logs — adding and subtracting powers.

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